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ON GOOD ET0L FORMS

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Abstract. This paper continues the study of ET0L forms and good E0L forms done by Maurer, Salomaa and Wood. It is proven that binary very complete ET0L forms exist, good synchronized ET0L forms exist and that no propagating or synchronized ET0L form can be very complete.

1. Introduction

Maurer, Salomaa, and Wood introduced in [1] and [2] the notion of E0L and ET0L forms and their interpretations. An ET0L form F defines a family of ET0L systems $\mathcal{G}(F)$ which are "structurally" similar and a family of languages $\mathcal{L}(F)$, namely those languages generated by systems in $\mathcal{G}(F)$. In [3] Maurer, Salomaa and Wood study good and complete E0L forms. Many of the theorems in [3] are trivially valid for ET0L forms as well as for E0L forms but they will not appear in this paper. In this paper we show that in contrast to what is the case for E0L forms, there exist good synchronized ET0L forms. Finally we show that (as for E0L forms) there exist binary vomplete (short for very complete) ET0L forms and that no propagating or synchronized ET0L form is vomplete.

2. Definitions and basic results

We follow the definitions of ET0L forms and interpretations given in [1], [2] and [3]. E0L forms will not be defined explicitly. We can consider an E0L form as an ET0L form with one table.

Definition 2.1. An ET0L *system* (or n-ET0L *system*) is an $(n+3)$-tuple $G = (V, \Sigma, P_1, \ldots, P_n, S)$, where $n \geq 1$, V is an alphabet, $\Sigma \subseteq V$ is the terminal alphabet, $S \in V - \Sigma$ is the *start symbol*. For all i, $1 \leq i \leq n$, P_i is a finite set of pairs (A, α) with $A \in V$ and $\alpha \in V^*$ such that for each $A \in V$ at least one such pair is in

P_i. The elements (A, α) are called *rules* or *productions* and are usually written $A \underset{P_i}{\rightarrow} \alpha$ or just $A \rightarrow \alpha$. The sets P_i are called *tables*.

Definition 2.2. Let $G = (V, \Sigma, P_1, \ldots, P_n, S)$ be an n-ET0L system. For words $x = A_1 A_2 \cdots A_m$ and $y = \alpha_1 \alpha_2 \cdots \alpha_m$ with $A_i \rightarrow \alpha_i$ in P_j for $1 \leq i \leq m$ and some P_j we write $x \underset{P_j}{\Rightarrow} y$ or $x \Rightarrow y$. $\Rightarrow^+$ (and $\Rightarrow^*$) are the transitive (and reflexive) closure of $\Rightarrow$. The language generated by G is

$$L(G) = \{x \in \Sigma^* : S \Rightarrow^* x\}.$$

Notice that in contrast to the usual definition of ET0L systems, the start symbol cannot be a terminal.

Definition 2.3. Let $G = (V, \Sigma, P_1, \ldots, P_n, S)$ be an n-ET0L system. For a word x, $|x|$ denotes the length of x and $\mathrm{Alph}(x)$ denotes the minimal alphabet such that $x \in \mathrm{Alph}(x)^*$. For all $1 \leq i \leq n$ let $\mathrm{maxr}(P_i) = \max\{|\alpha| : A \rightarrow \alpha \text{ in } P_i\}$, and let $\mathrm{maxr}(G) = \max\{\mathrm{maxr}(P_i) : 1 \leq i \leq n\}$. A symbol $B \in V$ is *reachable* (from S) if $S \Rightarrow^* \alpha B \beta$ for some words $\alpha, \beta \in V^*$. G is *reduced* if each $B \in V$ is reachable. G is *separated* if for all productions $A \rightarrow \alpha$ in $P_1, \ldots, P_n$ $\alpha \in (V - \Sigma)^*$ if $A \in \Sigma$ and $\alpha \in \Sigma \cup (V - \Sigma)^*$ otherwise. G is *propagating* if for all productions $A \rightarrow \alpha$ in $P_1, \ldots, P_n$, $\alpha \neq e$, the empty word. G is *synchronized* if, for all $a \in \Sigma$, $a \Rightarrow^+ \alpha$ implies $\alpha \notin \Sigma^*$. G is *short*, if not for all P_i, $A \rightarrow \alpha \in P_i$ implies $|\alpha| \leq 2$. Finally, G is *binary* if each rule in $P_1, \ldots, P_m$ is one of the forms $A \rightarrow e$, $A \rightarrow a$, $A \rightarrow B$, $A \rightarrow BC$, $a \rightarrow A$, where $a \in \Sigma$ and $A, B, C \in V - \Sigma$.

Definition 2.4. An ET0L *form* (or n-ET0L *form*) is an ET0L system $F = (V, \Sigma, P_1, \ldots, P_n, S)$. An ET0L system $G = (V', \Sigma', P'_1, \ldots, P'_n, S')$ is an *interpretation of* F (mod μ), $G \lhd F(\mu)$, or simply $G \lhd F$, if μ is a substitution defined on V satisfying (i)–(v):

(i) $\mu(A) \subseteq V' - \Sigma'$ for $A \in V - \Sigma$,
(ii) $\mu(a) \subseteq \Sigma'$ for $a \in \Sigma$,
(iii) $\mu(\alpha) \cap \mu(\beta) = \emptyset$ for any symbols $\alpha \neq \beta$,
(iv) for all $1 \leq i \leq n$, $P'_i \subseteq \mu(P_i)$, where

$$\mu(P_i) = \{(A', \alpha') \in V' \times V'^* : A' \in \mu(A), \alpha' \in \mu(\alpha) \text{ for some } A \in V, \alpha \in V^* \text{ such that } A \rightarrow \alpha \in P_i\},$$

(v) $S' \in \mu(S)$.

The family of ET0L *systems generated by* F, denoted $\mathscr{G}(F)$, is:

$$\mathscr{G}(F) = \{G : G \lhd F\}.$$

The family of languages generated by F, denoted $\mathscr{L}(F)$, is:

$$\mathscr{L}(F)=\{L(G): G\in\mathscr{G}(F)\}.$$

Since an ETOL form is an ETOL system, and conversely, we will allow ourselves to use the term "form" in the rest of this paper.

Definition 2.5. Two ETOL forms F_1 and F_2 are *equivalent* if $L(F_1)=L(F_2)$ and *form equivalent* if $\mathscr{L}(F_1)=\mathscr{L}(F_2)$.

The following lemmata are either contained in [2] or are a slight modification of some in [2].

Lemma 2.6. *Let $F=(V,\Sigma,P_1,\ldots,P_m,S)$ be an* ETOL *form and $F'=(V',\Sigma',P'_1,\ldots,P'_m,S')\lhd F(\mu)$. Then for each derivation*

$$x_0\underset{P'_{i1}}{\Longrightarrow}x_1\underset{P'_{i2}}{\Longrightarrow}\cdots\underset{P'_{ik}}{\Longrightarrow}x_k\quad \textit{in } F',$$

$\mu^{-1}(x_0)\underset{P_{i1}}{\Longrightarrow}\mu^{-1}(x_1)\underset{P_{i2}}{\Longrightarrow}\cdots\underset{P_{ik}}{\Longrightarrow}\mu^{-1}(x_k)$ *is a derivation in F.*

Lemma 2.7. *For all* ETOL *forms F a form equivalent reduced* ETOL *form F' can be constructed.*

Because of this lemma we will always assume the forms in this paper to be reduced.

Lemma 2.8. *Let $F=(V,\Sigma,P_1,\ldots,P_m,S)$ and $\bar{F}=(V\cup\bar{V},\Sigma,\bar{P}_1,\ldots,\bar{P}_m,S)$ with $V\cap\bar{V}=\emptyset$. If there are integers $k_1,k_2,\ldots,k_m$, such that* (a) *holds if and only if* (b) *holds for some i, then $\mathscr{L}(F)=\mathscr{L}(\bar{F})$.*

(a) $A\in V$, $A\underset{\bar{P}_{i_1}}{\Longrightarrow}x_1\underset{\bar{P}_{i_2}}{\Longrightarrow}x_2\underset{\bar{P}_{i_3}}{\Longrightarrow}\cdots\underset{\bar{P}_{i_t}}{\Longrightarrow}x_t$, $x_t\in V^*$, *and* $x_j\notin V^*$ *for* $1\le j<t$.

(b) $t=k_i$, $i_1=i_2=\cdots=i_t=i$, $A\to x_t$ *in* P_i, $x_j\in\bar{V}^+$ *for* $1\le j<t$, $A\to x_1$ *in* $\bar{P}_i$, *and* $x_{j-1}\underset{\bar{P}_i}{\Longrightarrow}x_j$ *for* $1<j\le t$.

Definition 2.9. Let F be an ETOL form and $\mathscr{F}$ a family of languages. We call F *$\mathscr{F}$-complete* or *complete for $\mathscr{F}$* if $\mathscr{L}(F)=\mathscr{F}$; if $\mathscr{F}$ is the family of ETOL languages, then we simply call F *complete* instead of ETOL-complete. We call F *good*, if for each ETOL form $\bar{F}$ with $\mathscr{L}(\bar{F})\subseteq\mathscr{L}(F)$ and ETOL form F' exists such that $F'\lhd F$ and $\mathscr{L}(F')=\mathscr{L}(\bar{F})$. F is called *bad* if it is not good. F is called *vomplete* (short for very complete) if it is complete and good.

3. Results

Most of the theorems on good E0L forms in [3] are easily shown to be valid for ET0L forms as well. Properties which are different for E0L and ET0L forms are related to synchronization. There are two "canonical" ways to synchronize an ET0L form. The first is to introduce a marked version of the terminals and make these new nonterminals and then change the productions by marking the terminals and add $a' \to a$, $a \to N$, $N \to N$ to all tables for all terminals a. N is a new nonterminal. The second, which has no counterpart in the E0L case, is to add a new table consisting of the productions $a' \to a$, $a \to N$, $N \to N$, $A \to N$ for terminals a and nonterminals A. $a \to N$, $N \to N$ is added to the rest of tables. The following lemma and theorem show different properties for synchronized E0L and ET0L forms.

Lemma 3.1. *The synchronized* ET0L *form*

$$F = (\{S, a, N\}, \{a\}, \{S \to SS; a \to N; N \to N\}, \{S \to a; a \to N; N \to N\}, S)$$

generates no nonempty finite languages.

Proof. Immediate. □

All synchronized E0L forms generating nonempty languages generate finite nonempty languages. This is used to prove that no good synchronized E0L form exists [3, Theorem 2.6]. The following theorem shows that good synchronized ET0L forms exist. Surprisingly enough the form shown to exist generates finite languages only!

Theorem 3.2. *The synchronized* ET0L *form*

$$F = (\{S, a, N\}, \{a\}, \{S \to S; a \to N; N \to N\}, \{S \to a; a \to N; N \to N\}, S)$$

is good.

Proof. $\mathscr{L}(F)$ consists of all nonempty finite languages consisting of single letter words. Let F' be an arbitrary ET0L form such that $\mathscr{L}(F') \subseteq \mathscr{L}(F)$. Assume $L(F') = \Sigma = \{a_1, \ldots, a_n\}$ and let $\mathscr{D}_{F'}$ denote the family of languages $\mathscr{D}_{F'} = \{L(G) : G \lhd F'(\mu), \mu(a) = \{a\}$ for all $a \in \Sigma\}$. Since the languages in $\mathscr{L}(F')$ consist of singletons $\mathscr{L}(F')$ can be characterized by:

$$L \in \mathscr{L}(F')$$

if and only if there exist $\eta \in \mathscr{D}_{F'}$ and finite substitution μ on η such that

(1) $\mu(a) \neq \emptyset$ for all a in η,

(2) $\mu(a) \cap \mu(b) = \emptyset$ for all $a \neq b$ in η, and

(3) $L \bigcup_{a \in \eta} \mu(a)$.

Because of this characterization it suffices to show that there exists an interpretation $\bar{F}$ of F such that the corresponding $\mathscr{D}_{\bar{F}}$ equals $\mathscr{D}_{F'}$. Let $K = \bigcap_{\eta \in \mathscr{D}_{F'}} \eta$. K denotes the set of symbols in Σ, which occur in all languages of $\mathscr{D}_{F'}$. Let R be the relation on Σ defined as follows: $(a, b) \in R$ if and only if for all $\eta \in \mathscr{D}_{F'}$ and $a \in \eta$ imply $b \in \eta$. Define $\bar{R}(a)$, for $a \in \Sigma$, to be the smallest set Q such that $a \in Q$, and $(b, c) \in R$ and $b \in Q$ imply $c \in Q$. Let $\bar{R}(M)$, for $M \subseteq \Sigma$, denote $\bigcup_{a \in M} \bar{R}(a)$.

$\mathscr{D}_{F'}$ can then be characterized by:

$$\eta \in \mathscr{D}_{F'} \quad \text{if and only if} \quad K \subseteq \eta \text{ and } \bar{R}(\eta) = \eta.$$

Without loss of generality we can assume that $K = \{a_1, \ldots, a_k\}$ for some $k \leq n$. Finally let $\bar{R}(a_i) = \{a_i^{(1)}, a_i^{(2)}, \ldots, a_i^{(k_i)}\}$ with $a_i = a_i^{(1)}$. Note that $\bar{R}(K) = K$.

Construct the ETOL form $\bar{F} = (V, \Sigma, P_1, P_2, S_1) \lhd F(\mu)$ as follows:

(i) $V = \Sigma \cup \{S_1, S_2, \ldots, S_k\} \cup \{N\} \cup \bigcup_{k \leq i \leq n} \{S_i^{(1)}, S_i^{(2)}, \ldots, S_i^{(k_i)}\}$.

(ii) P_1:
$$
\begin{array}{ll}
S_i \to S_{i+1} & \text{for } 1 \leq i < k, \\
S_k \to S_1 | S_{k+1}^{(1)} | S_{k+2}^{(1)} | \cdots | S_n^{(1)}, & \\
S_i^{(j)} \to S_i^{(j+1)} & \text{for } k < i \leq n \text{ and } 1 \leq j < k_i, \\
S_i^{(k_i)} \to S_1 & \text{for } k < i \leq n, \\
a_i \to N & \text{for } 1 \leq i \leq n, \\
N \to N. &
\end{array}
$$

(iii) P_2:
$$
\begin{array}{ll}
S_i \to a_i & \text{for } 1 \leq i \leq k, \\
S_i^{(j)} \to a_i^{(j)} & \text{for } k < i \leq n,\ 1 \leq j \leq k_i, \\
a_i \to N & \text{for } 1 \leq i \leq n, \\
N \to N. &
\end{array}
$$

(iv) For all $a \in \Sigma$, $\mu(a) = \{a\}$.
$\mu(S) = V - \Sigma$.

From the construction of $\bar{F}$ it follows that if we define $\mathscr{D}_{\bar{F}} = \{L(G) . G \lhd \bar{F}(\mu),\ \mu(a) = \{a\}$ for $a \in \Sigma\}$ then $\mathscr{D}_{\bar{F}} = \mathscr{D}_{F'}$ and therefore $\mathscr{L}(\bar{F}) = \mathscr{L}(F')$. □

Corollary 3.3. *If F is an* ETOL *form generating a nonempty finite language consisting of single letter words only, then there exists an integer k such that $\mathscr{L}(F)$ equals the family of all finite languages of size at least k and consisting of single letter words only.*

Although there exist good synchronized ETOL forms the following shows that no synchronized ETOL form is vomplete.

Lemma 3.4. *Let $F = (\{S, a\}, \{a\}, \{S \to a;\ a \to aa\}, S)$. Then no synchronized* ETOL *form F' is form equivalent to F.*

Proof. Assume that F' is an arbitrary synchronized ETOL form and that $\bar{F} = (\bar{V}, \{a\}, \bar{P}_1, \ldots, \bar{P}_m, \bar{S}) \lhd F'$ with $L(\bar{F}) = L(F)$ and let

$$\bar{S} \underset{\bar{P}_{i_1}}{\Longrightarrow} x_1 \underset{\bar{P}_{i_2}}{\Longrightarrow} \cdots \underset{\bar{P}_{i_q}}{\Longrightarrow} x_q = a^4$$

Proof. F in Lemma 3.7 is an E0L form. $\square$

To prove completeness in the E0L case, we have to show that for an arbitrary E0L form we can reduce the length of the right-hand sides of the productions below a certain limit without changing the family of languages generated. For ET0L forms we have to be able to reduce the number of tables as well. The next theorem shows that this is indeed possible. Similar theorems are proven in [2] for synchronized ET0L forms.

Theorem 3.11. *Given an* ET0L *form F, a form equivalent* 2-ET0L *form F′ can be constructed.*

Proof. Let $F = (\{A_1, \ldots, A_n\}, \{A_1, \ldots, A_t\}, P_1, \ldots, P_m, A_n)$. We construct a form equivalent 2-ET0L form $F' = (V, \{A_1, \ldots, A_t\}, P'_1, P'_2, A_n)$ as follows:

(i) $V = \{A_1, \ldots, A_n\} \cup \{A_i[j] : 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{N\}$.

(ii) P'_1 consists of the productions:

$$\left.\begin{array}{l} A_i \to A_i[1], \\ A_i[j] \to A_i[j+1],\ 1 \leq j < m, \\ A_i[m] \to N, \end{array}\right\}\ 1 \leq i \leq n,$$

$$N \to N.$$

(iii) P'_2 consists of the productions:

$$A_i \to N, \quad 1 \leq i \leq n,$$

$$N \to N,$$

$$A_i[j] \to \alpha, \quad \text{where } A_i \to \alpha \text{ is a production in } P_j,\ 1 \leq i \leq n,\ 1 \leq j \leq m.$$

From the construction it follows easily that $A \to \alpha \in P_j$ if and only if

$$A \in \{A_1, \ldots, A_n\}, \qquad A \underset{P'_1}{\Longrightarrow} \alpha_1 \underset{P'_1}{\Longrightarrow} \alpha_2 \underset{P'_1}{\Longrightarrow} \cdots \underset{P'_1}{\Longrightarrow} \alpha_j \underset{P'_2}{\Longrightarrow} \alpha,$$

where $\alpha_i \notin \{A_1, \ldots, A_n\}^*$, $1 \leq i \leq j$.

Therefore $L(F) = L(F')$.

Now let $\bar{F} \lhd F(\mu)$ be an arbitrary interpretation. We will prove that there exists an $\bar{F}' \lhd F'(\mu')$ such that $L(\bar{F}) = L(\bar{F}')$ and therefore $\mathscr{L}(F) \subseteq \mathscr{L}(F')$. Let $\bar{F} = (\{B_1, \ldots, B_p\}, \{B_1, \ldots, B_q\}, \bar{P}_1, \ldots, \bar{P}_m, B_p)$. We construct $\bar{F}' = (\bar{V}, \{B_1, \ldots, B_q\}, \bar{P}'_1, \bar{P}'_2, B_p) \lhd F'(\mu')$ such that $L(\bar{F}') = L(\bar{F})$ as follows:

(i) $\bar{V} = \{B_1, \ldots, B_p\} \cup \{B_i[j] : 1 \leq i \leq p, 1 \leq j \leq m\} \cup \{N\}$.

(ii) $\bar{P}'_1$ consists of the productions:

$$\left.\begin{array}{l} B_i \to B_i[1], \\ B_i[j] \to B_i[j+1],\ 1 \le j < m, \\ B_i[m] \to N, \end{array}\right\}\ 1 \le i \le p,$$

$$N \to N.$$

(iii) $\bar{P}'_2$ consists of the productions:

$$B_i \to N,$$

$$N \to N,$$

$$B_i[j] \to \beta, \quad \text{where } B_i \to \beta \text{ is a production in } \bar{P}_j,\ 1 \le i \le p,\ 1 \le j \le m.$$

(iv) $$\left.\begin{array}{l} \mu'(A_k) = \mu(A_k), \\ \mu'(A_k[j]) = \{B_i[j] \mid B_i \in \mu(A_k)\}, \end{array}\right\}\ 1 \le k \le n,\ 1 \le j \le m.$$

$$\mu'(N) = \{N\}.$$

As above it follows easily that $L(\bar{F}) = L(\bar{F}')$. That $\bar{F}' \lhd F'(\mu')$ is as clear.

Now let $G' \lhd F'(\eta')$ be an arbitrary interpretation. We will prove that there exists a $G \lhd F(\eta)$ such that $L(G') = L(G)$ and therefore $\mathscr{L}(F') \subseteq \mathscr{L}(F)$ which will complete the proof of the theorem.

Let $G' = (W', \Sigma, T'_1, T'_2, S)$. We construct $G = (W, \Sigma, T_1, T_2, \ldots, T_m, S)$ as follows:

(I) $W = \bigcup_{1 \le i \le n} \eta'(A_i)$.

(II) $\eta = \eta'|_{\{A_1,\ldots,A_n\}}$.

(III) $c \to \gamma$ is a production in T_j, $1 \le j \le m$ if and only if $c \in W$, $\gamma \in W^*$ and

$$c \underset{T'_1}{\Longrightarrow} \gamma_1 \underset{T'_1}{\Longrightarrow} \gamma_2 \underset{T'_1}{\Longrightarrow} \cdots \underset{T'_1}{\Longrightarrow} \gamma_j \underset{T'_2}{\Longrightarrow} \gamma.$$

Note that $W' = W \cup \bigcup_{1 \le i \le n; 1 \le j \le m} \eta'(A_i[j])$ and $\Sigma \subseteq W$.

Since $G' \lhd F(\eta')$ we get from Lemma 2.6 and (III) above that

$$(*) \qquad \eta'^{-1}(c) \underset{P'_1}{\Longrightarrow} \eta'^{-1}(\gamma_1) \underset{P'_1}{\Longrightarrow} \cdots \underset{P'_1}{\Longrightarrow} \eta'^{-1}(\gamma_j) \underset{P'_2}{\Longrightarrow} \eta'^{-1}(\gamma) \quad \text{if } c \to \gamma \text{ in } T_j.$$

Then $\gamma_i \notin W^*$, $1 \le i \le j$. Therefore $L(G') = L(G)$. To prove that $G \lhd F(\eta)$ we have to check points (i) through (v) in Definition 2.4. (i), (ii), (iii), and (v) follow from (II). To prove (iv) assume that $c \to \gamma$ is in T_j, $A_i = \eta^{-1}(c)$ for some $1 \le i \le n$, and $\delta = \eta^{-1}(\gamma)$. From (*) above we get

$$A_i \underset{P'_1}{\Longrightarrow} A_i[1] \underset{P'_1}{\Longrightarrow} A_i[2] \underset{P'_1}{\Longrightarrow} \cdots \underset{P'_1}{\Longrightarrow} A_i[j] \underset{P'_2}{\Longrightarrow} \delta$$

which implies that $A_i \to \delta$ is a production in P_j (in F). □

Theorem 3.12. *Given an* ETOL *form F a form equivalent short* ETOL *form F' can be constructed.*

Proof. The proof is very similar to one in [1] for E0L forms. Let $F = (V, \Sigma, P_1, \ldots, P_m, S)$. If $\text{maxr}(F) \leq 2$ then F is already short. If $\text{maxr}(F) > 2$ then it suffices to show that we can construct a form equivalent ETOL form $\bar{F} = (\bar{V}, \Sigma, \bar{P}_1, \ldots, \bar{P}_m, S)$ such that for some i, $\text{maxr}(\bar{P}_i) + 1 = \text{maxr}(P_i) = \text{maxr}(F)$ and for $j \neq i$ $\text{maxr}(\bar{P}_j) = \max\{\text{maxr}(P_j), 1\}$. Now let $\text{maxr}(P_i) = \text{maxr}(F) > 2$. We construct $\bar{F}$ as follows:

(I) $\bar{V} = V \cup \{N\} \cup \{B^{(p)}, C^{(p)} : p \in P_i\}$.

(II) $\bar{P}_i$ consists of the productions:

$$\left.\begin{array}{l} A \to B^{(p)}, \\ B^{(p)} \to \alpha, \\ C^{(p)} \to N. \end{array}\right\} \text{ if } |\alpha| \leq 2 \text{ and } p: A \to \alpha \text{ is a production in } P_i,$$

$$\left.\begin{array}{l} A \to B^{(p)}C^{(p)} \\ B^{(p)} \to A_1 \cdots A_{k-1} \\ C^{(p)} \to A_k \end{array}\right\} \text{ if } \alpha = A_1 \cdots A_k \text{ for some } k > 2 \text{ and } p: A \to \alpha \text{ is a production in } P_i.$$

$$N \to N.$$

(III) $\bar{P}_j = P_j \cup \{B^{(p)} \to N : p \in P_i\} \cup \{C^{(p)} \to N : p \in P_i\} \cup \{N \to N\}$ for $j \neq i$, $1 \leq j \leq m$.

By using Lemma 2.8 with $k_i = 2$ and $k_j = 1$ for $j \neq i$ we get that $\mathcal{L}(F) = \mathcal{L}(\bar{F})$. That $\text{maxr}(\bar{P}_i) = \text{maxr}(P_i) - 1$ and $\text{maxr}(\bar{P}_j) = \max\{\text{maxr}(P_j), 1\}$ for $j \neq i$ is clear. □

Theorem 3.13. *Given an* ETOL *form F a form equivalent short* 2-ETOL *form F' can be constructed.*

Proof. Immediate from the proofs of Theorems 3.11 and 3.12. □

Theorem 3.14. *The binary* 2-ETOL *form* $F = (\{a, S,\} \{a\}, \{a \to S; S \to S\}, \{a \to S; S \to e|a|S|SS\}, S\})$ *is complete.*

Proof. Completeness follows from [2, Theorem 5.5]. Given an arbitrary ETOL form F' we can construct a form equivalent ETOL form F'_1, which is reduced and separated using [2, Lemma 4.1 and 4.2]. Then using the constructions occurring in the proofs of Theorems 3.11 and 3.12 we obtain a form equivalent 2-ETOL form F'_2 which is reduced and binary. F'_2 must then be an interpretation of F, so F is therefore a good ETOL form. □

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